Northwest and
Alaska
Fisheries Center

National Marine
Fisheries Service

U.S. DEPARTMENT OF COMMERCE

NWAFC PROCESSED REPORT 83-11

An Age Structured Stochastic Recruitment and Management Model for the Pacific Whiting Fishery:

Establishment of model stability and robustness via sensitivity analysis

July 1983

NOTICE

An Age Structured Stochastic Recruitment and Management Model for the Pacific Whiting Fishery: Establishment of Model Stability and Robustness Via Sensitivity Analysis

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1 INTRODUCTION

The Getz-Swartzman model is an age structured stochastic model which has been applied to four fish stocks, the South African anchovy, New England yellowtail flounder, Georges Bank haddock (Getz and Swartzman 1981) and the Pacific whiting (Francis et al. 1982$ Swartzman et al. 1983). In all of these applications, estimates of model parameters have been assumed to be known with complete certainty. As is usually the situation in simulation modelling, parameter estimation in these applications was based on incomplete and sometimes scanty data sets. Little, if any, information was known concerning the sampling or underlying biological variability associated with the data sets. It is important to note that even though the Getz-Swartzman model is a stochastic model and consequently predicts expected values of stock and yield and their variances, the model predictions do not reflect uncertainty associated with the values assumed for parameters. All model predictions are based on point estimates of the parameters. Particularly in the Pacific whiting application in which model results may influence real life management decisions, the effect of parameter uncertainty on model behavior must be assessed. It has to be established that the model predictions are robust and stable and the model will not produce vastly different results when parameters are varied. A commonly used approach to explore the influence of various inputs on model behavior is sensitivity analysis. This paper reports on an extensive sensitivity analysis of the Pacific whiting version of the Getz-Swartzman model.

Sensitivity analysis has received widespread useage and the term has been loosely applied to practically any modelling exercise in which model inputs are varied and their influence on model behavior is assessed. This general operational definition of sensitivity analysis has resulted in many different methodologies being used with little attention paid to the dependancy of sensitivity results on the particular method chosen. Methods currently in use range in complexity from the classical partial derivative proposed by Tomovic (1963) to approaches involving parametric and nonparametric statistics (Gardner et al. 1981$ Spear and Hornberger 1980) and frequency domain analysis (Cukier et al. 1978).

With the current non-standard use of sensitivity analysis in mind, Rose (1982) and Rose and Swartzman (1981) reviewed and compared the various methods proposed in the literature. Particular attention was paid to how methods would fare when applied to large, complex simulation models such as the Getz-Swartzman model.

Rose (1982) details four problems that arise with the sensitivity analysis of large models and which methodologies must address. These problems are:

1)The computational requirements of the analysis. The computer costs and time of repeated runs of a large model can quickly become excessive.

2) The possibility of important parameter interaction effects. Interaction effects occur when the reponse of the model to varying values of a parameter depends on the values chosen for other parameters. Methods incapable of incorporating interaction effects can produce sensitivity conclusions for a given parameter that depend on the values assumed for other parameters. Figure 1 illustrates this using a simple two parameter example. The value of a model output variable is observed for two values of PARAM1 with PARAM2=k and PARAM2=k'. Based on the response of the output variable to the changed values of PARAM1 for PARAM2=k, PARAM1 would be concluded as "unimportant" whereas if PARAM1 was changed with PARAM2=k', PARAM1 would be inferred as "important".

3) The size and shape of the parameter space for which sensitivity conclusions are valid. Many methods only explore a small region in the parameter space (size) and make unrealistic assumptions about the relative variabilities of different parameters (shape). Complex models can respond in very different and unpredictable (non-linear) ways to variations in parameters dependent on the size and shape of the parameter space explored.

4) The complication of interpreting sensitivity conclusions caused by large models offering many potential output variables on which to base the sensitivity analysis. Additionally, not only will sensitivity conclusions vary from output to output variable, but the conclusions for a given output variable will vary with time in the simulation.

Using problems (1) - (4) as criteria, Rose (1982) evaluated the various proposed methods concluding that random sampling approaches and fractional factorial designs were the best suited for use on large models. Since the two methods can generate very different sensitivity conclusions when applied to the same model (Rose 1982), we believe that both must be used and sensitivity conclusions compared for corroboration before interpretation of the results can proceed.

This paper is organized as follows. First the random sampling and fractional factorial methods of sensitivity analysis are briefly described in general terms. This is followed by a section which summarizes the structure of the Pacific whiting version of the Getz-Swartzman model. Next the details involved in applying the sensitivity methods to the model are discussed. The final two sections present interpretations of the sensitivity analyses of both the simulation model and the management algorithm and conclusions about the robustness and stability of both.

2 SUMMARY OF SENSITIVITY ANALYSIS METHODS

Fractional factorial designs with analysis of variance (ANOVA) and a type of random sampling termed Latin Hypercube sampling (LHS) with partial rank correlation (PRC), were the methods deemed best suited for use on large by Rose (1982). A third method, which we call individual parameter perturbation (IPP), is a very commonly used method and, as will be discussed, is useful in conjunction with the fractional factorial design method in establishing the importance of parameter interaction effects. The methods are two step procedures. The first step is the generation or sampling of parameter values to be used in model runs and the second step is the statistical analysis of the resulting output which provides quantities on which parameters can be ranked by importance.

2.1 Fractional Factorial Designs

In factorial designs, runs are made such that all 'levels' of a parameter are combined with all 'levels' of every other parameter. The simplest case is having two levels, a nominal value and either plus or minus a constant percentage from the nominal value, which for d parameters results in 2^d runs. A fractional factorial design involves running only a fraction (1/2 to some power c) of the total possible factorial runs and is commonly denoted as 2^{d-c} (Box et al. 1978). The information lost as a result of not making all the runs appears as the estimated effects of various parameter combinations on the output being aliased or inseparable from the available runs. The best that can be done is an estimate of the sum of the effects of the aliased parameter combinations. Judicious selection of which runs to make enables individual parameter effects to be aliased only with high order interaction effects and low order interaction effects with interactions of moderate order. By assuming that higher order interactions are negligible, individual parameter effects may be estimated. All low order interaction effects may not be separable in some designs but usually their relative importance as a group can be determined. Box et al. (1978) define the resolution R of a fractional factorial design as one in which no K parameter interaction is aliased with any other combinations containing R-K parameters.

2.2 Analysis of the Output from Fractional Factorial Designs

Steinhorst et al. (1978) apply analysis of variance (ANOVA) to the values of the output variable from a fractional factorial design and use the F-values to rank parameters. Since there is no random component to model output the F-values represent empirical quantities, not probability statements. Parameters can therefore be equivalently ranked on the magnitude of their sum of squares from the ANOVA.

2.3 Random Sampling

The most common random sampling procedures assume the parameters are uncorrelated. A probability distribution is assumed for each parameter and for each run every parameter is sampled from its distribution. To gain as much information as possible from the fewest runs, parameters which sufficiently cover the parameter space must be selected in an efficient manner. For a limited number of runs, simple random sampling can result in a clustering of parameter values leaving gaps in the coverage of the parameter space and a redundancy in the information contained in some runs. This redundance may be eliminated through the use of some form of stratified sampling.

Conceptually, stratified sampling ensures good coverage with little redundance by dividing the parameter space into sections and sampling from each section with certain probabilities. McKay et al. (1976$1979) go a step further and propose Latin Hypercube sampling (LHS) which divides each parameter range into intervals. LHS requires the same number of intervals for each parameter as model runs. The intervals for each parameter are randomly assigned to model runs (i.e., each interval appears only once) and for each run values are selected from the intervals. In application, the intervals for each parameter are chosen such that every interval contains the same amount of area under the assumed probabilility distribution. This way one can sample from an interval assuming a uniform distribution over the interval.

2.4 Analysis of the Output from Random Sampling

Mckay et al. (1976) advocate the use of the partial rank correlation coefficient (PRC) between the values of the output variable and each parameter over runs as a measure of sensitivity. The premise is that the greater the correlation between the output variable and a parameter over runs, the greater is the controlling influence of that parameter on model behavior. The rationale for using the rank transformed values of the output variable and parameters is to lessen the influence that a few outlier values can have on the correlation coefficients and consequently the sensitivity conclusions.

2.5 Individual Parameter Perturbation

Individual parameter perturbation (IPP) is a straightforward, intuitive approach to sensitivity analysis. First a run in which all parameters are kept at their nominal values is made, referred to as the standard run. Parameters are then varied one-at-a-time by some percentage from their nominal values. The importance of parameters are determined by the magnitude of the change of the output variable from each run with the corresponding value in the standard run.

IPP is probably the most commonly used method despite the fact that it cannot incorporate parameter interaction effects. Unlike fractional factorial designs and LHS, which vary parameters simultaneously, IPP singly perturbs parameters which prevents obtaining any information on interactions. The IPP method is used in this paper to represent the sensitivity conclusions obtained if interactions are ignored. Since the fractional factorial design with ANOVA explicitly estimates interaction effects and IPP ignores them, the degree of similarity between the two methods indicates the effect of interactions on sensitivity conclusions.

3 SUMMARY OF THE GETZ-SWARTZMAN MODEL

The Getz-Swartzman model is fully documented in Getz and Swartzman (1981). The model can be summarized as follows:

a) The number of individuals in each of n age classes at time k in the simulation is characterized by a probability vector Pi(k), i=1,...,n, where the jth element pij(k), j=1,...,m, represents the probability that the actual numbers of individuals in the ith age class is in the interval:

$$[\ XMAXi(k) * (j-1)/m\ ,\ XMAXi(k) * j/m\ ] \qquad (1)$$

The maximum population level, XMAXi(k) for each age class i is calculated by accruing natural and fishing mortality via a Beverton-Holt type analsis:

$$XMAX\langle i+1\rangle(k) = \exp\{\ -(ALPHi + Qi*V(k)*L)\} * XMAXi(k) \qquad (2)$$

where ALPHi, Qi, V(k), and L are, respectively, the natural mortality rate, catchability coefficient, fishing effort, and length of the fishing season as a fraction of the total season. The maximum recruitment, XMAX1(k), is constant over time (k = 1,2,...) and is determined from spawner-recruit data as discussed below in (c). The probability vector Pi(k) is carried forward in time as the age group passes into the next year class:

$$P\langle i+1\rangle(k+1) = Pi(k) \qquad i=1,...,n-2 \qquad (3)$$

The only exception is the probability vector of the largest size class, Pn(k+1), which consists of individuals not only of age n-1 but also older individuals (see Getz and Swartzman 1981 for details).

b) The fecund stock biomass level in the fishery at time k, STOCK(k), is characterized by a probability vector PS(k) with ms elements. The jth element represents the probability that the actual fecund stock biomass level is in the jth range:

$$[\ SMAX * (j-1)/ms\ ,\ SMAX * j/ms\ ] \qquad (4)$$

The probability vector PS(k) is obtained by preforming a convolution on the n age class probability vectors Pi(k), i=1,...,n. The elements of each Pi(k) vector are assigned midpoints values of the intervals defined in equation (1). In a similar manner, the elements of PS(k) are assigned the midpoints values of the intervals defined in equation (4). To convert from numbers of individuals in the Pi(k) to the units of fecund stock biomass in PS(k), the midpoint values of each age class i are multiplied by a constant average weight Wi and the fraction of individuals which are sexually mature Ci.

The expected value and variance of STOCK(k) are calculated using the expressions:

$$E(STOCK(k)) = PS_j(k) * \{(j-.5)/ms\} * SMAX$$

$$V(STOCK(k)) = PS_j(k) * \{(j-.5)/ms\} * SMAX$$

Analogous expressions for the expected value and variance of YIELD(k) based on the Pi(k) are also computed using the amount of biomass from each age class which is removed due to fishing in equation (2).

c) The probability vector P1(k+1), not obtainable from equation (3), characterizing the probability distribution of new recruits at time k, is determined from the spawning stock using a stock-recruit probability transition matrix approach. The elements, t_{ij}, i=1,...,m and j=1,...,ms, of the probability transition matrix (T) represent the probability with which a fecund stock level in the jth biomass range will result in the number of recruits falling into the range:

$$[\ XMAX1 * (i-1)/m\ ,\ XMAX1 * (i/m)\ ]$$

Thus, in general, if newly spawned individuals are only recruited to the fishery at age r, it follows that:

$$P1(k+1) = T * PS(k+1+r)$$

d) Adaptation of the Getz-Swartzman model to the Pacific whiting fishery required specifying that recruitment to the fishery take place at age 3 years and that there be n=8 age classes, m=6 subdivisions in each age class, and ms=8 stock subdivisions. A list of parameters and their values used in the simulation are presented in Table 1. Additonally, we have interpreted results from Bailey (1981) that the level of recruitment entering the whiting population each year is correlated with the water temperature on the spawning grounds during larval development. To incorporate this information, water temperatures on the spawning grounds were categorized as either warm or cold. Two spawner-recruit probability transition matrices were used corresponding to the levels and variablilites of recruitment in 'warm years' (TW) and the levels and variabilites in 'cold years' (TC).

4 APPLICATION OF SENSITIVITY METHODS TO THE MODEL

The objectives of the sensitivity analysis are twofold: First to determine the effects of parameter variation on model behavior and on the results of the management algorithm$ Second, to determine what aspects of the model control model behavior. The approach taken was to use the methods described in the previous section to investigate the behavior of the model and then to use the results from the sensitivity analysis of the model to investigate aspects of the management algorithm. Details and results of the sensitivity analysis of the model are presented in the following section (section 5). The management algorithm analysis is then presented in section 6.

4.1 Choice of Parameters and Model Runs

Table 1 presents the 23 parameters, their definitions, and nominal and plus and minus 10# values which were used in the sensitivity analyses. There are several aspects of the parameters listed in Table 1 that deserve further discussion. First, two of the parameters, TC and TW, are not the usual scalar quantities but rather are matrices. The interpretation of these probability transition matrices is based on the discretization of stock and recruitment into subdivisions. Each element of the matrix (t_{ij}) gives the probability of observing the number of recruits in subdivision i given the stock is in subdivision j. There are two transition matrices which represent the different recruitment for warm versus cold water temperature on the spawning grounds.

Variation of the transition matrices for sensitivity analysis can be done in several ways. The most detailed approach would be to vary each element of each matrix. This results in 60 individual parameters which is clearly too many for the analysis. Reducing the number of parameters associated with the matrices is not only justified due to the number of parameters involved but also based on how the matrices were estimated. In the application of the model to the Pacific whiting, the matrices were estimated from a single spawner-recruit data set implying that both matrices were estimated as a single quantity. The estimation procedure used was to first examine the spawner-recruit data and specify the maximum possible recruitment level, XMAX1, and the maximum possible stock level, SMAX. The values of XMAX1 and SMAX determine the scale of the ordinate (number of recruits) and abscissa (stock biomass level) respectively, for the transition matrices. Next the elements of the matrices were estimated from the spawner-recruit data on a column by column basis. We were able to specify the mean and standard deviation (SD) of each column of the warm year matrix (TW) and cold year matrix (TC). A column in a transition matrix, t_{ij}, $i=1,\ldots,n$, corresponds to the probability vector giving the probabilities of observing each of the n subdivisions of recruitment given the stock is in the jth stock subdivision. The elements in each column were determined such that they resulted in the desired mean and SD of recruitment. One may conclude that based on the estimation procedure described above both matrices should be treated as a single parameter and only be allowed to vary together since both came from the same spawner-recruit data set. The problem with this is that the transition matrices contain a lot of information and treating them as a single parameter severely limits how we can vary them to assess model sensitivity. Since the primary objective here was sensitivity analysis and determining model stability, it is not necessary that all values chosen for parameters be completely realistic. Therefore, we opted for a compromise between treating each element of the transition matrices as parameters (a total of 60 parameters) and treating both matrices as a single parameter. We decided to treat XMAX1, SMAX, TC, and TW as four separate parameters. Knowing that independently varying the four parameters is not completely realistic, we also investigated the effect on model behavior of using a different set of values for the transition matrices which could have realistically arisen from the original spawner-recruit data set.

The values of the elements in the two parameters, TC and TW, were varied in the sensitivity analyses based on the mean and SD of each column. Perturbation by +10# from the nominal value implies that the probabilities in each column are adjusted such that the mean number of recruits is increased by 10# keeping the SD unchanged (see Table 1a). There are many possible ways to alter the probabilities which would achieve this. The method used here is to arbitrarily alter the probabilities by trial and error while satisfying two constraints:

1) The 'unimodal pattern' is preserved. This means that beginning at the top or bottom the probabilities in a column must increase or decrease. For example, the probabilities cannot increase, decrease, and then increase again forming a 'bimodal pattern'.

2) In a column, the only zero probabilities that can be altered are those originally adjacent to non-zero probabilities.

A final notable aspect of the parameters listed in Table 1 is that the fraction mature of 3 year olds, C3, cannot be perturbed upward 10# since the nominal value of 1.0 is an upper bound.

Unless otherwise noted, all model runs are for 47 years using historical water temperatures and fishing efforts as driving variables. The dynamics of the model for this historical run when all parameters are set to their nominal values, referred to as the standard run, are shown in Figures 2a and b.

4.2 Selection of Model Ouput Variables

The behavior of the model is characterized by the predicted mean and variance of stock and yield biomasses. (Note: In this paper the term 'stock' will refer to fecund stock.) A total of eleven output variables were used and are defined in Table 2. The output variables of mean stock at times 8, 26, and 44 years and mean yield at times 34 and 46 years are used to investigate the time dependency of sensitivity conclusions (problem (4)). SDEV and YDEV aggregate over time the values of mean stock and yield, summing the absolute change in their values from those in the standard run. SDEV is based on the sum from times 7 to 47 years which eliminates the begining of the simulation when arbitrary initial conditions may influence model behavior. YDEV only includes the years 33 to 47 since these were the times a fishery operated on the Pacific whiting. Sensitivity conclusions based on SDEV and YDEV reflect the behavior of the model over the entire run. The remaining four output variables, SVWITH, YVWITH, SVBETW, and YVBETW, attempt to characterize the effect parameters have on the variability of predicted values of stock and yield. The first two are based on the within year variances predicted from the probability distributions of stock and yield while the second two are the between year variances of stock and year over the entire run.

4.3 Implementation of the Methods

A 2^{23-17} R=4 fractional factorial design requiring 64 runs was used with parameters perturbed by either plus or minus 10# as defined in Table 1. Two complete analyses, notated SIGN-A and SIGN-B, were done with SIGN-B identical to SIGN-A except for the directions of perturbations being reversed (see Table 3). This was done to see if the direction of perturbation chosen for parameters has an effect on sensitivity conclusions.

The choice of whether to perturb parameters plus or minus 10# in the SIGN-A and SIGN-B runs was not done randomly but rather the directions chosen for some parameters was dictated by the directions of other parameters. This was done to preserve the pattern of values among certain parameters. Specifically the nominal values of the catchabilities, Qi, i=1,8, had the pattern of Qi less than Q<i+1> for i=1,6 and Q8 less than Q6 and Q7. Another pattern among the nominal values involved the natural mortalities where ALPHi is greater than ALPH<i+1> for i=1,7. Examination of the perturbed values of parameters in Tables 1 and 3 for the SIGN-A and SIGN-B runs shows that in both these patterns were preserved.

LHS was implemented assuming uniform distributions for each parameter with endpoints of plus and minus 10# of the nominal values. Each distribution was divided into 200 equal intervals resulting in 200 model runs as recommended by Rose (1982) and Iman et al. (1980). Parameters TC and TW were generated in a manner consistent with their plus and minus 10# values used in the fractional factorial design. Using TW as an example, for each run a random number (K) between -1 and +1 was obtained. The value of TW used in the run was then calculated from the tuned matrix (TWT), +10# matrix (TWU), and -10# matrix (TWL) as follows:

1) for K in the interval (-1, 0):

$$TW = (1-K)*TWT + (K)*TWL$$

2) for K in the interval (0, +1):

$$TW = (1-K)*TWT + (K)*TWU$$

IPP was applied to the model for both the SIGN-A and SIGN-B sets of directions of perturbation.

4.4 Measurement of the Similarity Between Methods

To facilitate the many comparisons of sensitivity conclusions generated by the different methods it is necessary to define a measure of the degree of similarity between the sensitivity conclusions from two methods. The measure of similarity we chose was the sum of the absolute deviation in ranks (SADR) between the top 10 ranked parameters according to either method:

$$SADR = \sum_{\substack{\text{for all} \\ P_j \text{ ranked} \\ \text{in top10 by} \\ \text{either method}}} \left| \text{rank1}(P_j) - \text{rank2}(P_j) \right|$$

where rank1(P_j) and rank2(P_j) are the ranks given P_j by the two methods.

5 RESULTS AND DISCUSSION

The results from the sensitivity analyses are presented in two parts. The first part is a comparison among the various methods using the SADR to make certain that they give similar sensitivity conclusions. Also included is a discussion of the importance of parameter interaction effects. The second part is a summary and interpretion of the results from all the methods and several additional model runs which provide information on what controls model behavior.

5.1 Consistency Among Methods

Table 4 presents the SADR of the top 10 ranked parameters comparing the fractional factorial designs and IPP for the SIGN-A and SIGN-B runs and LHS. The output variable on which sensitivity conclusions in Table 4 are based is SVWITH. The results were similar for all the output variables except SDEV and YDEV. It is very encouraging that for 9 out of 11 output variables the three methods generate similar sensitivity conclusions. This does not necessarily mean that the conclusions are the same for the different output variables but rather that for a given output variable, the methods agree as to which parameters are important. These results are in agreement with the evidence that indicates parameter interaction effects are not important for these output variables. In both the SIGN-A and SIGN-B fractional factorial designs there were no aliased two parameter interaction effects ranked more important than 4th. Furthermore, even the relatively important interaction ranked 4th only accounted for 5# of the total sum of squares in the ANOVA. Also encouraging is the agreement between the SIGN-A and SIGN-B versions of IPP and the fractional factorial designs. This indicates that the direction of perturbation chosen for parameters has little influence on the sensitivity conclusions.

The results from the remaining output variables of SDEV and YDEV are not as easily interpretable. Concentrating on the SIGN-B sets of runs, interaction effect 22 (defined in Table 5) is ranked first among all parameters for both SDEV and YDEV, accounting for approximately 20# of the total sum of squares for both variables. Table 6 shows SADR for SDEV and YDEV comparing the SIGN-B runs of the fractional factorial design and IPP and LHS. Whereas for the nine other output variables the three methods generated similar conclusions, this is not true for SDEV and YDEV. Given the importance of interaction effects as compared to the nine other variables this result is perhaps not surprising. What is surprising and completely counter-intuitive is that the conclusions from the fractional factorial design and IPP agree and both disagree with those from LHS. These results are exactly opposite to the conclusions reached by Rose (1982). Rose (1982), based on the application of these methods to a different model, reasoned that when interactions are important, the fractional factorial design and IPP would differ siqnificantly, since the fractional factorial design explicitly accounts for interaction effects in the ANOVA whereas IPP ignores interactions. Furthermore, Rose (1982) found that regardless of the importance of interactions, LHS and IPP always agreed to the same degree. The results from applying the methods to the Getz-Swartzman model show that when interactions are not important, LHS and IPP agree but that when interactions are important LHS and IPP differ significantly.

To provide the reader with an idea of how different the conclusions from the fractional factorial design, IPP, and LHS are, Table 7 compares the rankings given the top 3 parameters according to each method for YDEV. For comparison, included are the analogous results for YVBETW as the output variable.

Clearly for SDEV and YDEV interpretation of which parameters are important is dependent on which method is used and additional model runs would have to made in order to provide an explanation for this behavior. The appropriate procedure would be to apply another fractional factorial design to the parameters included in the interactions in Table 5 to determine which of the interactions are important. Although the information obtained from this further analysis would add greatly to our understanding of the model, we are limited by both time and resources and cannot pursue this any further. Therefore only the results from the nine output variables other than SDEV and YDEV will be discussed in the next section.

5.2 Interpretation of Sensitivity Results

The same five parameters were ranked as the top 5 most important parameters for all nine of the output variables. These parameters are XMAX1, SMAX, TC, TW, and ALPH1. In terms of the percent of the total sum of squares accounted for in the fractional factorial design, the magnitude of the PRC coefficient in LHS, and the change in the output variable from the standard run in IPP, the two top ranked parameters generally dominated over the remaining parameters. Also either XMAX1 or SMAX were usually ranked as the most important.

The almost complete domination of XMAX1 and SMAX was somewhat unexpected. By separating XMAX1, TC, and TW , a 10# increase in recruitment could be achieved via a 10# increase in XMAX1 or by altering the probabilities in each column of the transition matrices such that the mean is 10# higher. Table 8 shows that the effect of singly perturbing XMAX1 by 10# is equivalent to altering the transition matrices (TC and TW) for all the output variables except the four based on the variances of stock and yield. Interestingly, for SVWITH, YVWITH, SVBETW, and YVBETW, changing TC and TW simultaneously had much less effect on the output variables relative to the standard run than perturbing XMAX1. The values of the output variables when XMAX1 is changed indicate a substantial decrease in the variablility of predicted values of stock and yield as compared to the run with TC and TW changed and the standard run. This occurs despite the fact that by increasing XMAX1 the variance of each column of the transition matrices is increased as opposed to perturbing TC and TW in which the variance is maintained unchanged.

The great importance of XMAX1, SMAX, TC, and TW in determining model behavior coupled with the above results has important implications in parameter estimation. As discussed earlier, the transition matrices are typically estimated as single entities, with determination of the probability elements dependent on the choice of XMAX1 and SMAX. Yet neither XMAX1 or SMAX are point estimable, but rather represent values thought to be arbitrarily larger than any potentially observable values. An equally valid estimate of the matrices could have been made from the same spawner-recruit data set with XMAX1 and SMAX at different values. To illustrate this we went back to the original spawner-recruit data set and using an XMAX1 of 10# higher than the nominal value reestimated the probabilties in the columns of TC and TW (denoted TC′ and TW′ in Table 9). These estimated matrices differ from the ones used in the sensitivity analyses. They are not a variation of the nominal values of TC and TW but rather represent equally valid alternative estimates of the transition matrices obtained in the same manner as the nominal values but with a 10# larger XMAX1. The values of the output variables for a run with TC′ and TW′ are included in Table 8 and indicate that both valid sets of estimates of the matrices (TC and TW in the standard run and TC′ and TW′) would result in similar, but not identical, model predictions. In fact, the predicted variances of yield are more similar between the standard run and TC and TW perturbed by +10# than between the standard run and the run with TC′ and TW′.

Until this point, all discussion concerning parameter importance has been in relative terms. How do the actual values of stock and yield change when parameters are varied? Figures 3a and b show the time trajectories of stock and yield for the standard run and for XMAX1 perturbed by -10#, +10#, and +50#. The model seems quite stable. Similar patterns of stock and yield are evident in all the runs with higher values of stock and yield associated with higher values of XMAX1. Also, as XMAX1 is increased, the dynamics of stock become less and less variable. This is due to the higher levels of recruitment dictated by increased XMAX1 pushing stock into the upper end of the transition matrices where the highest levels of recruitment are obtained and stock size has no effect.

Notice that the patterns of stock are similar for the entire 47 years, not just for the years when fishing is operative. One explanation for the stability of the model during the years of fishing is that there is a feedback between stock and yield. The model computes yield from stock using the observed fishing efforts as a driving variable. The amount of yield removed each year is proportional to the effort multiplied by the biomasses of each age class comprising the stock. The values of yield in Figure 3b track the pattern of stock over time, high levels of stock are decreased by large yields and low levels of stock are decreased by small yields.

The similar patterns of stock for the years prior to fishing must be due to the time trace of temperatures (warm or cold) and the differences in recruitment in TC and TW. To investigate this, 47 year runs without any fishing were made for the same runs as in Figure 3 using the historical time trace of temperatures (Figure 4) and with warm and cold years switched (Figure 5). The controlling influence of temperature can clearly be seen as the switching of temperatures causes the pattern of stock in Figure 5 to be the 'mirror image' of the historical run in Figure 4.

In summary, the model is well behaved and quite stable under conditions of parameter uncertainty. Five parameters, XMAX1, SMAX, TC, TW, and ALPH1, are the most important parameters according to nine different output variables. Estimation of the probability transition matrices must be done with caution. The choice of XMAX1 and SMAX is arbitrary and different, equally valid estimates of the matrices can cause somewhat different model predictions of stock and yield variances. The pattern over time exhibited by stock and yield is controlled by the values of temperature and fishing efforts. The absolute levels of stock and yield are determined by the values of XMAX1, SMAX, TC, and TW.

6 SENSITIVITY ANALYSIS OF THE MANAGEMENT ALGORITHM

The management algorithm is fully described in Francis et al (1982) and Swartzman et al (in review). Conceptually the algorithm operates as a five year look-ahead forecasting recruitment into the future based on information concerning the water temperatures on the spawning grounds. The objectives of the algorithm are to protect the stock when it is in poor condition and temperatures do not appear conducive to stock improvement in the near future and to be able to utilize strong year classes in an efficient manner (Francis et al. 1982). These objectives are achieved by defining upper and lower stopping rules based on the biomass of 5+ year olds, termed the desireable stock. The lower stopping rule (SRL) is a value that desireable stock biomass cannot go below for any of the five years in the future. The upper stopping rule (SRU) is the optimal level that desireable stock should be equal to at the end of the fifth year. Fishing efforts for the five years are adjusted until the two stopping rules are satisfied. In general, efforts are adjusted to preferentially increase yield in the upcoming year at the expense of losing yield in the future.

The inputs of the model and management algorithm which were investigated are defined in Table 10. These include the five model parameters determined to be important in influencing model behavior, the upper and lower stopping rules of the algorithm, and the driving variable of temperature. Based on the results from the sensitivity analysis of the model, the simple IPP approach provides satisfactorily similar conclusions to LHS and fractional factorial design and so is used here since it requires the least number of model runs. Measurement of the performance of the algorithm is done by using the mean of yield, effort, and catch-per-unit-effort (CPUE) over a 47 year run with historical temperatures. All model parameters are kept at their nominal values unless otherwise stated.

The consequences of altering the stopping rules was reported in Table 6 in Francis et al. (1982). Portions of those results are reproduced in Table 11 as the standard run and Figures 6 and 7. The algorithm runs are noted as SRL/SRU in 1000 mt. The 293/293 run was concluded by Francis et al. (1982) to be the most desireable management run of the fishery. This run resulted in an average yield of 200,000 mt and had the highest average CPUE. Other runs had slightly higher average yields but at the expense of higher average efforts and less stable stock dynamics. Figures 6 and 7 compare the time trajectories of stock and yield biomass for the 293/293 run and a run in which 200,000 mt was removed each year as a constant quota. These figures demonstrate the stabilizing influence that the management algorithm has on model behavior. By its very definition, the algorithm restricts stock to be greater than the lower stopping rule and close to the upper stopping rule. We have already seen how fishing efforts as a driving variable control the dynamics of model behavior. With the management algorithm this control goes a step further in that efforts are continually adjusted dependent on the state of the stock and the upcoming effects of temperatures. The intensity of the stabilizing influence of the algorithm is demonstrated in Figure 8 in which two 293/293 runs are shown with the temperatures of one run switched (i.e., warm years to cold years and vice versa). In Figures 4 and 5 we saw how great the influence of switching temperatures was in affecting model behavior. Comparison of Figures 4 and 5 with Figure 8 shows that the dynamics of stock for both temperature sequences are more similar with the algorithm than with the historical runs. This indicates that the algorithm does have a very strong controlling influence on model behavior in the manner we would expect.

The effects of varying model parameters are presented in Table 11. The behavior of the algorithm is very stable. For all the parameters varied by plus and minus 10#, the algorithm results in average values of yield, effort, and CPUE which are nearly symmetric around the values in the standard run. As with model behavior, XMAX1 has the greatest effect on the management algorithm. Recall from the previous section that varying XMAX(1) versus TC and TW together had similar effects on predicted values of stock and yield but different effects on the predicted variances of these quantities. Table 11 also contains the results of the management algorithm for TC and TW simultaneously perturbed by -10#. Since evaluation of the algorithm only depends on the predicted values of stock and yield and the variability of these over the 47 years and not the predicted variances of these quantities, perturbation of XMAX1 and TC and TW have similar effects.

All of the results indicate that the algorithm has a great stabilizing influence on the model behavior. Also, although varying parameter values changed the results from the algorithm, in all runs the results changed in smooth, predictable ways. The model behavior never deviated very much from the standard run, even when the temperatures were switched.

7 CONCLUSIONS

There are other aspects of the model and management algorithm which could be investigated. For instance, in the algorithm, the manner by which efforts are incremented and decremented over the five year look ahead in the search to satisfy the stopping rules. However the purpose of this sensitivity analysis is not to alter the conclusions drawn from the model application but rather to establish the robustness and stabilility of the results. What we have established in this paper is that the Getz-Swartzman model as applied to the Pacific whiting fishery does not produce vastly different predictions as a result of variation in parameter values. The sensitivity analyses also showed that significant improvement in the accuracy of the predictions of stock and yield can come from increased confidence in spawner-recruit data from which XMAX1, SMAX, TC and TW are estimated.

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TABLE 1: The definitions, nominal values, and plus and minus 10% values of the 23 parameters investigated in the sensitivity analysis

parameter	definition	nominal value	-10% value	+10% value
SMAX	maximum stock size	1.5E+09	1.35E+09	1.65E+09
TC	cold year probability transition matrix	----------- see TABLE 1A ---------		
TW	warm year probability transition matrix	------------ see TABLE 1A ---------		
Q(1)	catchability coefficient for 3 year olds	0.0045	0.00405	0.00495
Q(2)	catchability coefficient for 4 year olds	0.0119	0.01071	0.01309
Q(3)	catchability coefficient for 5 year olds	0.0142	0.01278	0.01562
Q(4)	catchability coefficient for 6 year olds	0.0307	0.02763	0.03377
Q(5)	catchability coefficient for 7 year olds	0.0429	0.03861	0.04719
Q(6)	catchability coefficient for 8 year olds	0.0712	0.06408	0.07823
Q(7)	catchability coefficient for 9 year olds	0.0771	0.06939	0.08481
Q(8)	catchability coefficient for 10+ year olds	0.0700	0.06300	0.07700
C(1)	fraction of 3 year olds sexually mature	0.50	0.45	0.55
C(2)	fraction of 4 year olds sexually mature	0.75	0.675	0.825
C(3)	fraction of 5 year olds sexually mature	1.00	0.90	1.00
ALPH(1)	natural mortality of 3 year olds	0.270	0.243	0.297
ALPH(2)	natural mortality of 4 year olds	0.210	0.189	0.231
ALPH(3)	natural mortality of 5 year olds	0.200	0.180	0.220

TABLE 1: (continued)

parameter	definition	nominal value	-10%value	+10% value
ALPH(4)	natural mortality of 6 year olds	0.257	0.2313	0.2827
ALPH(5)	natural mortality of 7 year olds	0.357	0.3213	0.3927
ALPH(6)	natural mortality of 8 year olds	0.457	0.4113	0.5027
ALPH(7)	natural mortality of 9 year olds	0.557	0.5013	0.6127
ALPH(8)	natural mortality of 10 year olds	0.657	0.5913	0.7227
XMAX(1)	maximum number of recruits	2.4E+09	2.16E+09	2.64E+09

TABLE 1A: The nominal, -10%, and +10% values of TC and TW showing the mean and SD of each column. The midpoint values of each recruitment sub-category are: 2.2, 1.8, 1.4, 1.0, 0.6, 0.2 (xE09 individuals)

recruitment sub-category	TC-nominal (denoted TCT)					TW-nominal (denoted TWT)				
6-	0	0	0	0	0	0	0	0	.5	.1
5-	0	0	0	0	0	0	0	.05	.07	.15
4-	0	0	0	0	0	0	0	.15	.15	.15
3-	0	0	0	0	0	0	.2	.2	.23	.2
2-	0	.45	.6	.65	.68	.14	.6	.45	.35	.25
1-	1	.55	.4	.35	.32	.86	.2	.15	.15	.15
stock level sub-category	1	2	3	4	5-8	1	2	3	4	5-8
mean	.2	.38	.44	.46	.472	.256	.6	.8	.916	1.08
SD	0	.199	.196	.1908	.1866	.1388	.253	.429	.535	.627

recruitment sub-category	TC -10% (denoted TCL)					TW -10% (denoted TWL)				
6-	0	0	0	0	0	0	0	.001	.035	.056
5-	0	0	0	0	0	0	0	.029	.065	.104
4-	0	0	0	0	0	0	.0005	.1	.1	.145
3-	0	.005	.01	.001	.003	.001	.159	.205	.23	.205
2-	0	.335	.47	.559	.587	.079	.57	.465	.36	.27
1-	1	.66	.52	.44	.41	.92	.2705	.2	.21	.225
stock level sub-category	1	2	3	4	5-8	1	2	3	4	5-8
desired mean	.2	.342	.396	.414	.4248	.2304	.54	.72	.8244	.972
mean	.2	.338	.396	.4244	.437	.2324	.55	.7184	.822	.924
SD	0	.194	.2078	.1993	.1994	.1105	.259	.4	.517	.591

recruitment sub-category	TC +10% (denoted TCU)					TW +10% (denoted TWU)				
6-	0	0	0	0	0	0	0	.001	.055	.13
5-	0	0	0	0	0	0	0	.051	.092	.16
4-	0	0	0	0	0	0	.02	.163	.178	.16
3-	0	.005	.02	.025	.03	.001	.23	.305	.255	.21
2-	.05	.545	.67	.715	.75	.179	.59	.39	.32	.24
1-	.95	.45	.31	.26	.22	.82	.16	.09	.1	.1
stock level sub-category	1	2	3	4	5-8	1	2	3	4	5-8
desired mean	.22	.418	.484	.506	.5192	.2816	.66	.88	1.008	1.188
mean	.22	.422	.484	.506	.5242	.2724	.644	.8792	1.003	1.172
SD	.08	.2027	.198	.1917	.185	.155	.270	.407	.531	.623

TABLE 2: Definitions of the eleven output variables used in the sensitivity analyses. The summations of stock begin at time 7 since from years 1-6 arbitrary initial conditions influence model behavior. The summations of yield begin at the first year of fishing.

variable	definition
STOCK(8)	biomass of stock at time 8 years (1941)
STOCK(26)	biomass of stock at time 26 years (1959)
STOCK(44)	biomass of stock at time 44 years (1977)
YIELD(34)	biomass of yield at time 34 years (1967)
YIELD(46)	biomass of yield at time 46 years (1979)
SDEV	$\sum_{t=7}^{47} \lvert (\mathrm{STOCK}(t) - \mathrm{STOCK}(t)') / \mathrm{STOCK}(t)' \rvert$ where STOCK(t)' is the standard run
YDEV	$\sum_{t=33}^{47} \lvert (\mathrm{YIELD}(t) - \mathrm{YIELD}(t)') / \mathrm{YIELD}(t)' \rvert$ where YIELD(t)' is the standard run
SVWITH	average predicted variance of STOCK over the 47 years
YVWITH	average predicted variance of YIELD over the 47 years
SVBETW	variance of the predicted values of STOCK over the 47 years : $\mathrm{SVBETW} = \frac{\sum_{t=7}^{47} (\mathrm{STOCK}(t) - \overline{\mathrm{STOCK}})^2}{41}$ where $\overline{\mathrm{STOCK}} = \sum_{t=7}^{47} \mathrm{STOCK}(t)/41$
YVBETW	variance of the predicted values of YIELD over the 47 years : $\mathrm{YVBETW} = \frac{\sum_{t=33}^{47} (\mathrm{YIELD}(t) - \overline{\mathrm{YIELD}})^2}{15}$ where $\overline{\mathrm{YIELD}} = \sum_{t=33}^{47} \mathrm{YIELD}(t)/15$

TABLE 3: Directions of perturbations for the SIGN-A and SIGN-B sets of runs

parameter	SIGN-A	SIGN-B
SMAX	+	-
TC	-	+
TW	+	-
Q(1)	+	-
Q(2)	-	+
Q(3)	-	+
Q(4)	+	-
Q(5)	-	+
Q(6)	+	-
Q(7)	+	-
Q(8)	+	-
C(1)	-	+
C(2)	+	-
C(3)	-	-
ALPH(1)	+	-
ALPH(2)	-	+
ALPH(3)	-	+
ALPH(4)	+	-
ALPH(5)	+	-
ALPH(6)	+	-
ALPH(7)	+	-
ALPH(8)	+	-
XMAX(1)	-	+

TABLE 4: SADR of the top 10 parameters comparing the SIGN-A and SIGN-B runs of the fractional factorial design, IPP, and LHS. All sensitivity conclusions are based on SVWITH as the output variable.

		fractional factorial design		IPP		LHS
		SIGN-A	SIGN-B	SIGN-A	SIGN-B	
fractional factorial design	SIGN-A	0	15	11.5	20	15
	SIGN-B	---	0	15.5	10	10
IPP	SIGN-A	---	---	0	19	11
	SIGN-B	---	---	---	0	13
LHS		---	---	---	---	0

TABLE 5: The two parameter interactions contained in interaction effect #22

SMAX	x	ALPH(7)
TC	x	ALPH(1)
Q(1)	x	Q(7)
Q(3)	x	Q(5)
Q(8)	x	ALPH(5)
C(1)	x	XMAX(1)
C(2)	x	ALPH(3)
ALPH(2)	x	ALPH(6)

TABLE 6: SADR of the top 10 parameters comparing the SIGN-B runs of the fractional factorial design and IPP, and LHS. Results are presented for SDEV and YDEV as output variables.

OUPUT VARIABLE = SDEV

		fractional factorial design SIGN-B	IPP SIGN-B	LHS
fractional factorial design	SIGN-B	0	19	124
IPP	SIGN-B	---	0	105
LHS		---	---	0

OUPUT VARIABLE = YDEV

		fractional factorial design SIGN-B	IPP SIGN-B	LHS
fractional factorial design	SIGN-B	0	25	134
IPP	SIGN-B	---	0	131.5
LHS		---	---	0

TABLE 7: Rankings of the top 3 parameters comparing the SIGN-B runs of the fractional factorial design and IPP, and LHS. Results are presented for YDEV and YVBETW as output variables.

OUPUT VARIABLE = YDEV

rank	fractional factorial design SIGN-B	IPP SIGN-B	fractional factorial design SIGN-B	LHS	IPP SIGN-B	LHS
1	XMAX1	XMAX1	XMAX1	ALPH7	XMAX1	ALPH7
2	TW	TW	TW	TW	TW	TW
3	ALPH1	TC	ALPH1	ALPH8	TC	ALPH8

OUPUT VARIABLE = YVBETW

rank	fractional factorial design SIGN-B	IPP SIGN-B	fractional factorial design SIGN-B	LHS	IPP SIGN-B	LHS
1	TW	TW	TW	XMAX1	TW	XMAX1
2	XMAX1	XMAX1	XMAX1	TW	XMAX1	TW
3	ALPH1	ALPH1	ALPH1	ALPH1	ALPH1	ALPH1

TABLE 8: Values of all the output variables for the standard run, for XMAX(1) singly perturbed by +10%, and for TC and TW simultaneously perturbed by +10%.

	standard run	XMAX(1) +10%	TC and TW +10%
STOCK(8)	1.240E+09	1.310E+09	1.310E+09
STOCK(26)	.989E+09	1.080E+09	1.100E+09
STOCK(44)	.891E+09	.991E+09	.994E+09
YIELD(34)	.239E+09	.264E+09	.262E+09
YIELD(46)	.174E+09	.194E+09	.194E+09
SDEV	0.0	2.74	3.0
YDEV	0.0	1.66	1.62
SVWITH	42.80E+15	2.50E+15	34.50E+15
YVWITH	2.90E+15	.68E+15	2.94E+15
SVBETW	37.90E+15	9.00E+15	29.50E+15
YVBETW	7.78E+15	1.69E+15	9.32E+15

TABLE 9: Model parameters, algorithm parameters, and driving variables investigated in the sensitivity analysis of the management algorithm.

	parameter	definition
MODEL		
	SMAX	maximum stock size
	XMAX(1)	maximum number of recriuts
	TC	cold year transition matrix
	TW	warm year transition matrix
	ALPH(1)	natural mortality of 3 year olds
MANAGEMENT ALGORITHM		
	SRL	lower stopping rule
	SRU	upper stopping rule
DRIVING VARIABLES		
	TEMP	temperature on spawing grounds (warm or cold)

TABLE 10: Effects on the management algorithm of varying model parameters.

mean yield
(x E+09)

	-10%	standard run	+10%
XMAX(1)	.168	.200	.232
SMAX	.208	.200	.192
TC	.186	.200	.217
TW	.183	.200	.214
ALPH(1)	.208	.200	.192
TC and TW		---	.228

mean effort

	-10%	standard run	+10%
XMAX(1)	9.95	11.9	13.61
SMAX	12.18	11.9	11.42
TC	10.63	11.9	13.27
TW	11.21	11.9	12.31
ALPH(1)	12.39	11.9	11.21
TC and TW		----	13.35

mean CPUE

	-10%	standard run	+10%
XMAX(1)	16.8	16.8	16.8
SMAX	17.1	16.8	16.7
TC	17.3	16.8	16.4
TW	16.3	16.8	17.4
ALPH(1)	16.8	16.8	17.0
TC and TW		----	16.8

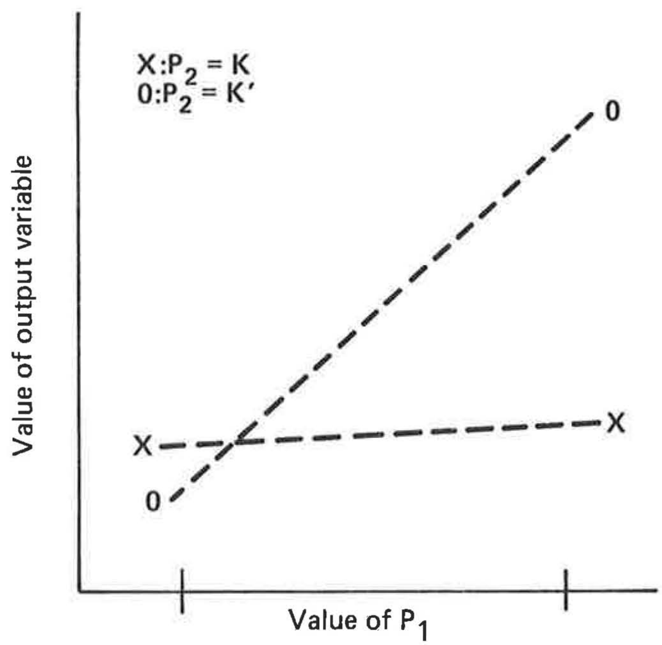

Figure 1. Hypothetical model responses depicting an interaction effect between parameters P1 and P2.

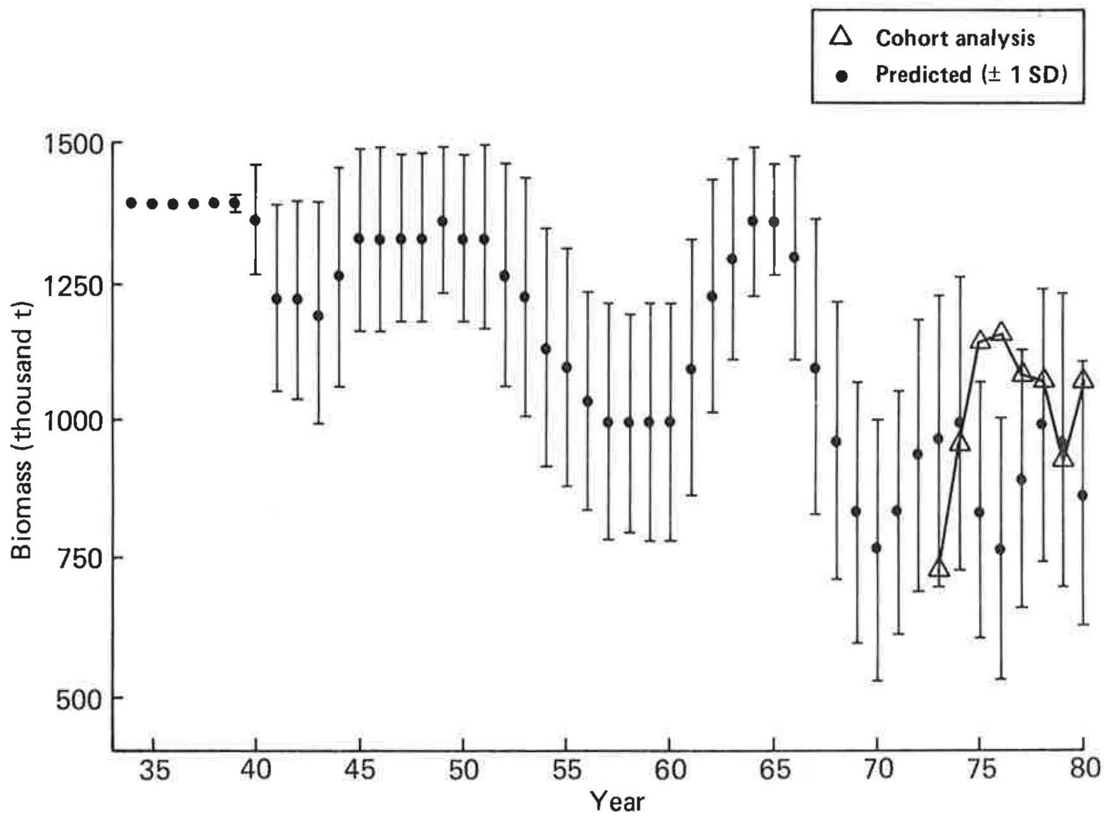

Figure 2A. Dynamics of STOCK, expected values and ±SD, for the 47 year standard run. All parameters are at their nominal values (Figure 5 in Francis et al. 1982).

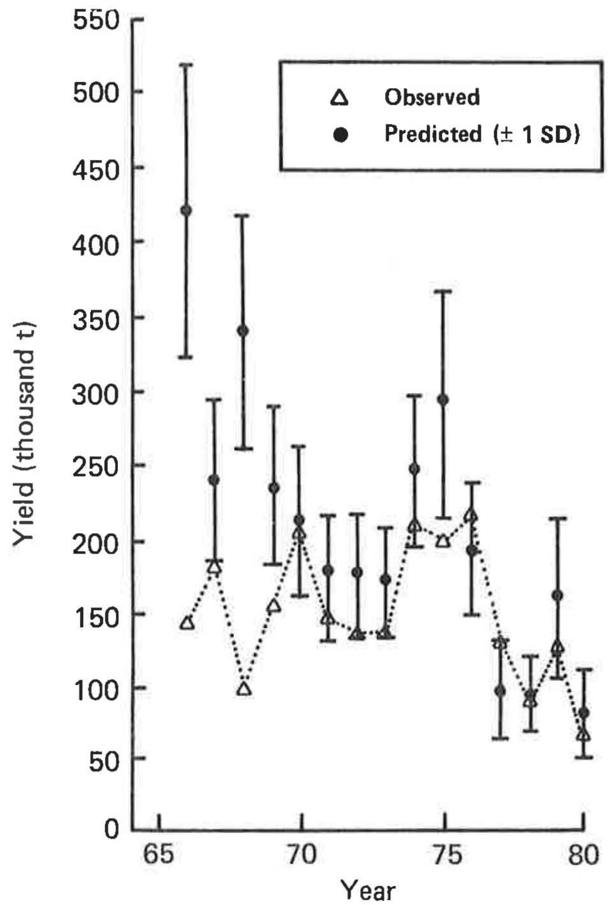

Figure 2B. Dynamics of YIELD, expected values and ±SD, for the 47 year standard run. All parameters are at their nominal values (Figure 6 in Francis et al. 1982).

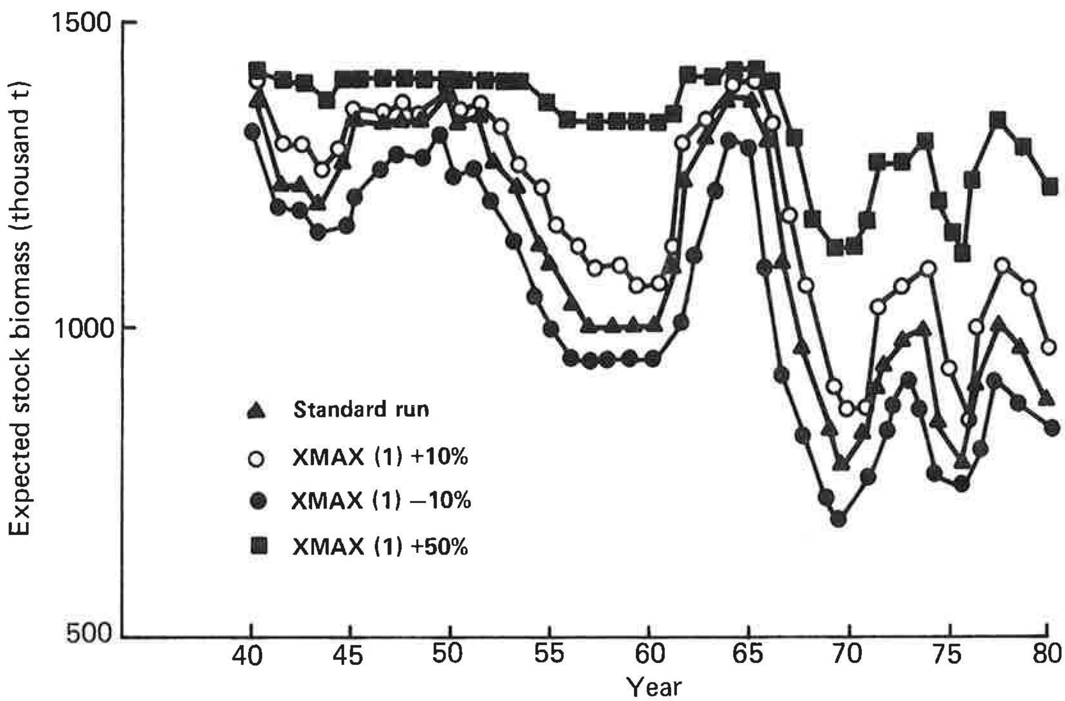

Figure 3A. Expected values of STOCK for the standard run and for XMAX(1) perturbed by -10%, +10%, and +50%.

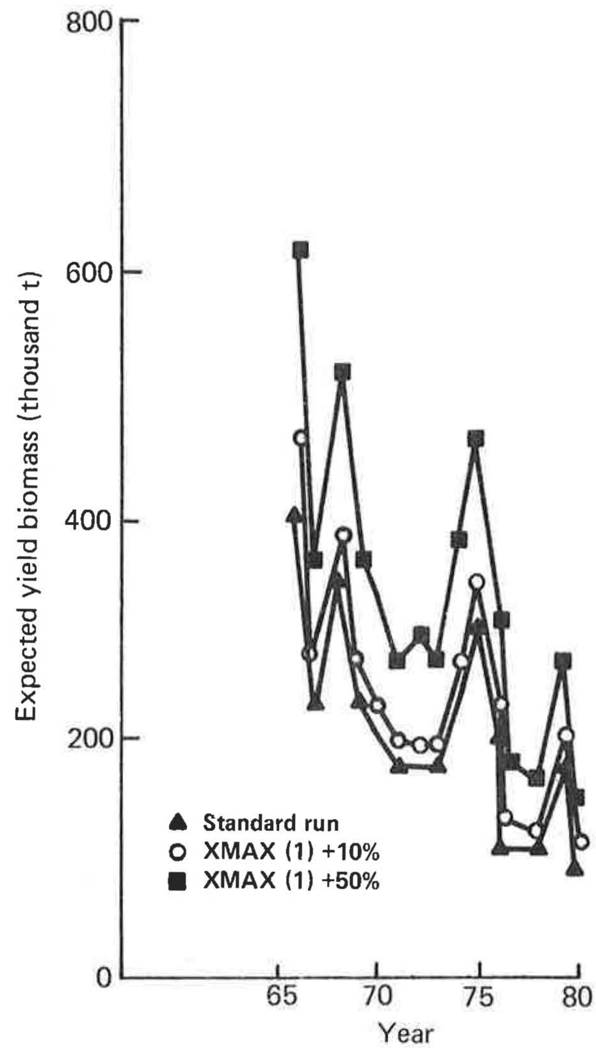

Figure 3B. Expected values of YIELD for the standard run and for XMAX(1) perturbed by -10%, +10%, and +50%.

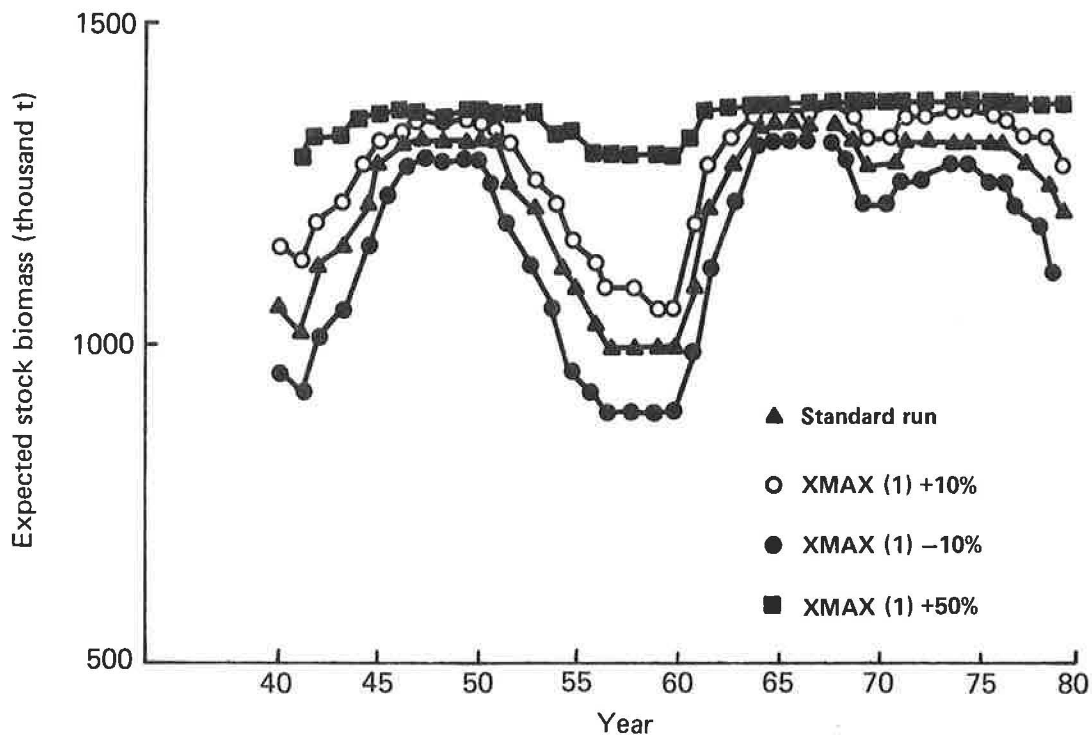

Figure 4. Same as Figure 3A but with no fishing efforts.

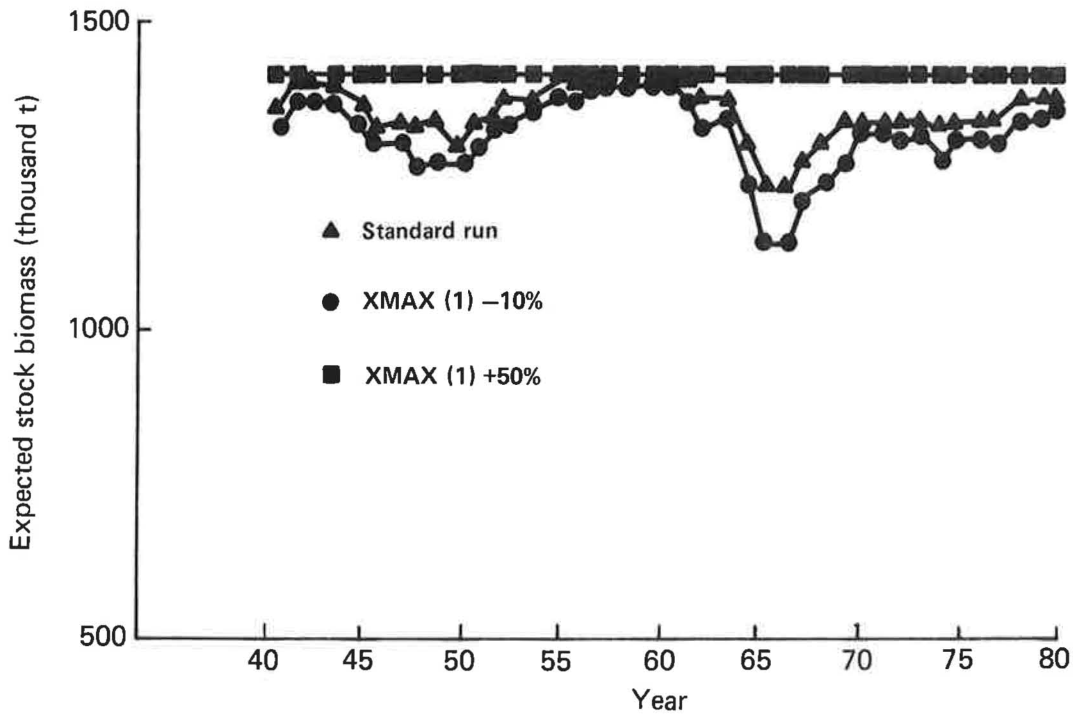

Figure 5. Expected values of STOCK for the standard run and for XMAX(1) perturbed by -10% and +50%. The values of the driving varaible of temperature are switched.

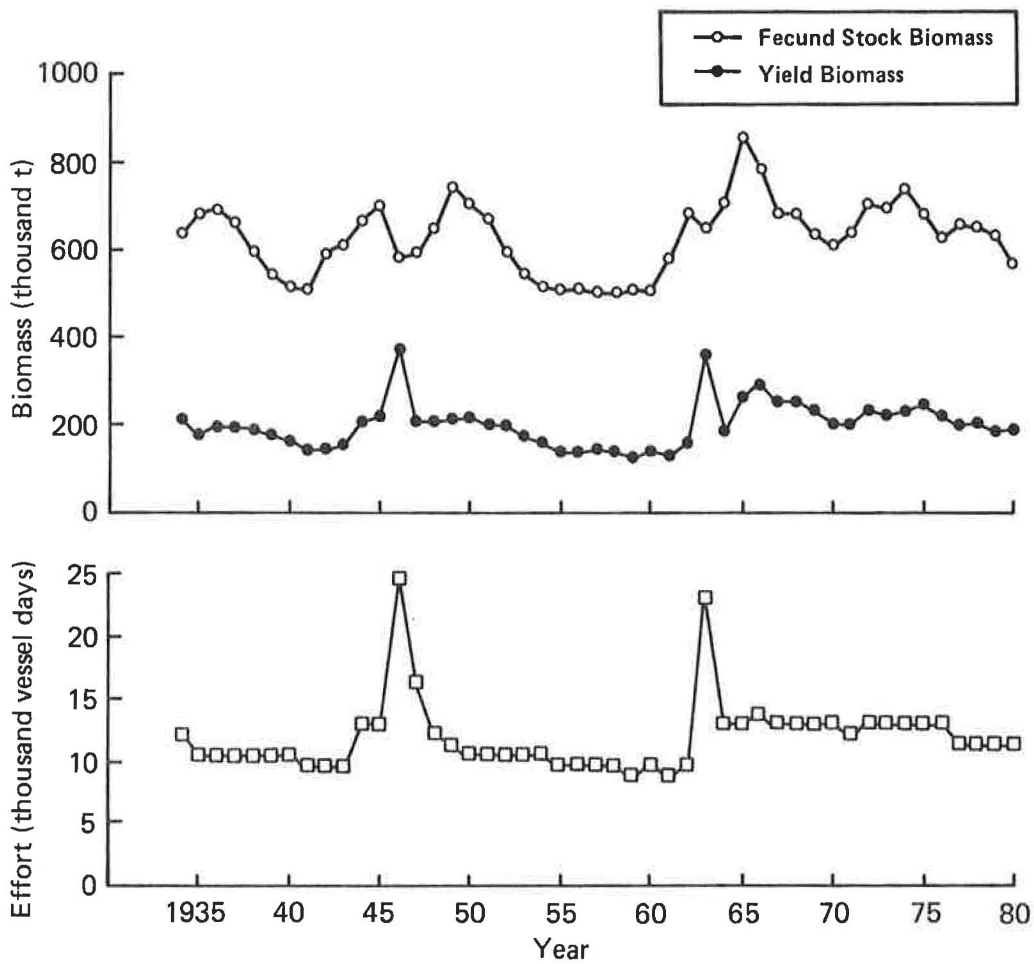

Figure 6. Expected values of STOCK and YIELD and the values of fishing efforts for the 47 year 293/293 management run. (Figure 14 in Francis et al. 1982).

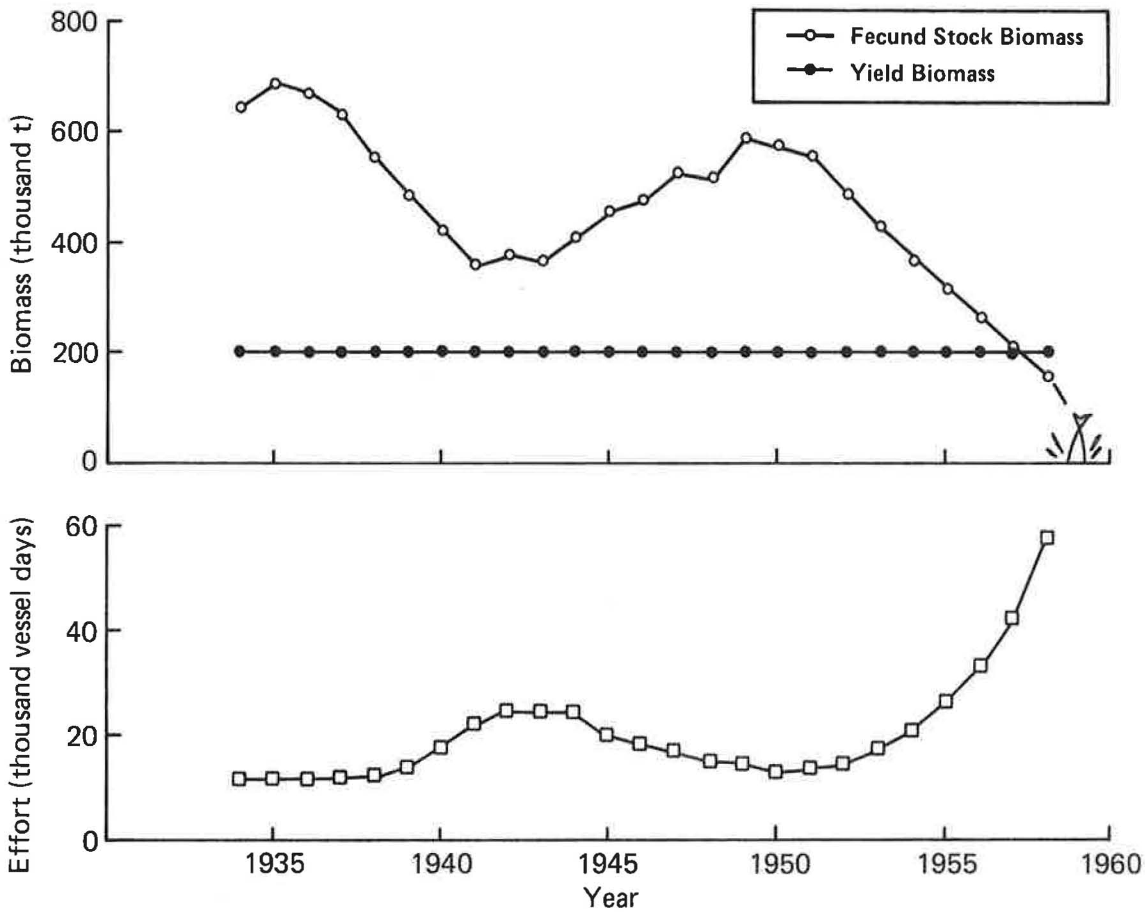

Figure 7. Expected values of STOCK and YIELD and the values of fishing efforts for a constant quota removal of 200,000 t. (Figure 15 in FRancis et al 1982).

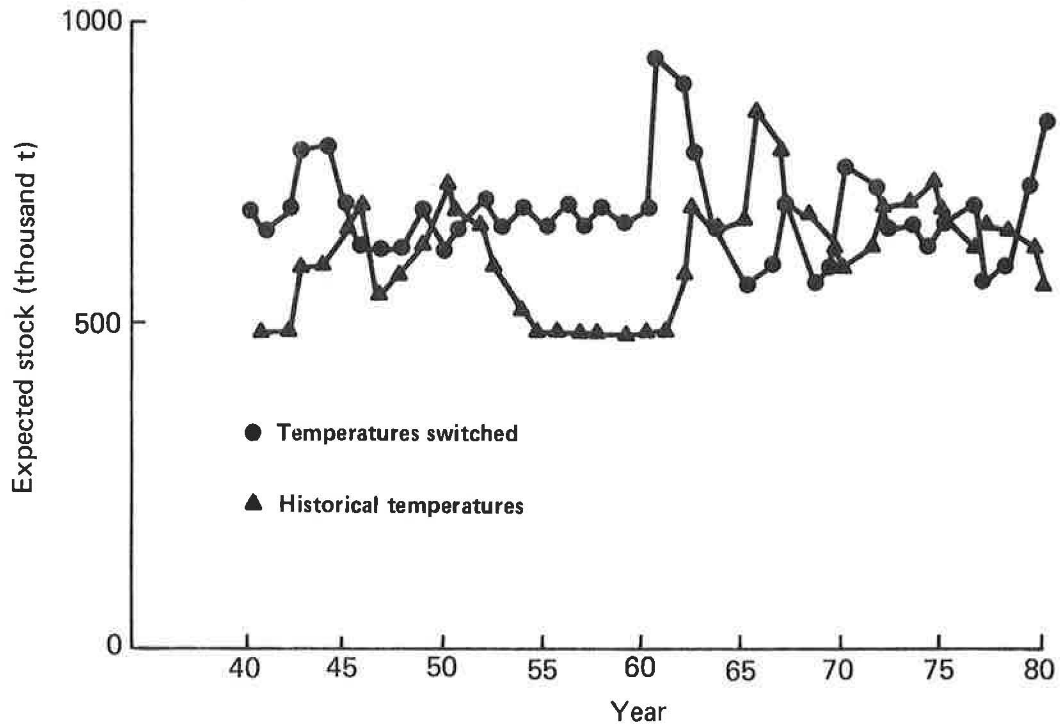

Figure 8. Expected values of STOCK for two 293/293 management runs in which one run has the temperatures switched.